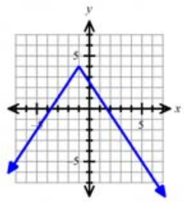
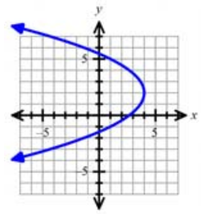
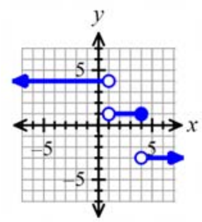
1) Multiple Select. Which relations below are functions? **Select all that apply.**

A) $\{(-9,-7),(-9,5),(1,2),(5,-2),(10,-2)\}$
B) $\{(4,-3),(7,-3),(0,2),(7,-3),(11,5)\}$
C) $x^2+y^2=25$
D) $y=-\sqrt{x+1}$
E) 
F) 
G) 

A function is a relationship between inputs (typically x) and outputs (typically y) in which there is only one output for any given input, e.g., only one y-value for any given x-value.

For a graph, this gives rise to the vertical line test: if you can construct a vertical line that touches two points of a curve, it is not a function.

Let's consider the seven sets/curves above:

A) The first two points in this set have the same x-value but different y-values. NOT a function.
B) The point $(7,-3)$ appears twice in the set. That's okay because the input, 7, has the same output, -3. Function.
C) If we graph the equation, we get a circle. Circles fail the vertical line test. NOT a function.
D) Any valid x-value ($x \geq -1$) will generate only one y-value. Function.
E) Passes the vertical line test. Function.
F) Fails the vertical line test. NOT a function.
G) Passes the vertical line test. Recall that an open circle at a point means that point is excluded from the set. A closed circle at a point means that point is included in the set. Function.

Conclusion: **B, D, E, and G are functions.**

For #2 – 3, evaluate each expression as indicated, and simplify.

2) $f(x)=3x^2+2x-3$; find $f(x+4)$.

$$f(x)=3x^2+2x-3$$
$$f(x+4)=3(x+4)^2+2(x+4)-3$$
$$=3(x^2+8x+16)+2x+8-3$$
$$=3x^2+24x+48+2x+8-3=\mathbf{3x^2+26x+53}$$

3) $g(x)=\frac{-2x^2+8}{x^3-2}$; find $g(-6)$.

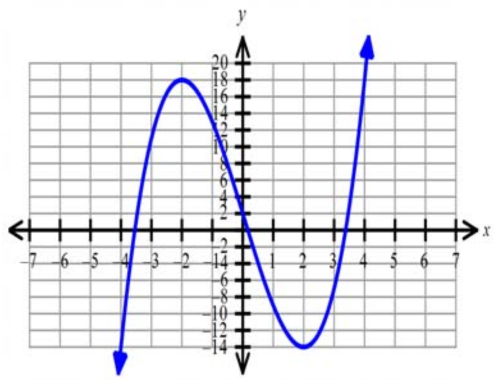
$$g(x)=\frac{-2x^2+8}{x^3-2}$$
$$g(-6)=\frac{-2(-6)^2+8}{(-6)^3-2}=\frac{-2(36)+8}{-216-2}=\frac{-72+8}{-218}=\frac{-64}{-218}=\mathbf{\frac{32}{109}}$$

For #4 – 5, find the following information for each function.

a) interval(s) increasing, in interval notation.
b) interval(s) decreasing, in interval notation.
c) Domain and Range, in interval notation.
d) coordinates of any relative extrema.

The Domain is the set of all valid x-values.
The Range is the set of all resulting y-values.
Extrema are the peaks and valleys of the curve. There are no extrema at endpoints.
A point where points immediately to the left and right are lower is a relative maximum.
A point where points immediately to the left and right are higher is a relative minimum.
A curve is neither increasing nor decreasing at extrema, nor at endpoints of the curve.
Multiple intervals can be joined with a "∪" symbol, indicating a "union" of intervals.

4)



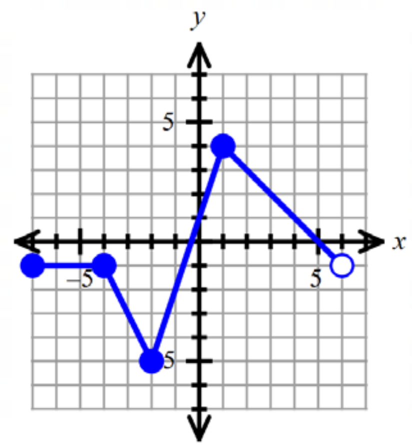
a) Increasing: $(-\infty, -2) \cup (2, \infty)$

b) Decreasing: $(-2, 2)$

c) Domain: $(-\infty, \infty)$ i.e., all real numbers, $\mathbb{R}$
Range: $(-\infty, \infty)$ i.e., all real numbers, $\mathbb{R}$

d) Relative extrema:
Relative maximum @ $(-2, 18)$
Relative minimum @ $(2, -14)$

5)



a) Increasing: $(-2, 1)$

b) Decreasing: $(-4, -2) \cup (1, 6)$

c) Domain: $[-7, 6)$
Range: $[-5, 4]$

d) Relative extrema:
Relative maximum @ $(1, 4)$
Relative minimum @ $(-2, -5)$

For #6 – 7, determine whether each function is even, odd, or neither.

A function is even if $f(x) = f(-x)$. Example: polynomials with only even powers of x, including constants since the power of x on a constant is zero. Even functions reflect over the y-axis.

A function is odd if $f(x) = -f(-x)$. Example: polynomials with only odd powers of x and no constants. Odd functions reflect over the origin $(0, 0)$.

6) $f(x) = 2x^2 + x^4$

$f(x)$ has only even powers of x, so it is an **even function**.

7) $h(x) = x^3 + x^2 + 3$

$h(x)$ has both even and odd powers of x, so it is **neither an even function nor an odd function**.

For #8 – 9, evaluate each piecewise function at the indicated value.

8) Find $g(-3)$ if $g(x) = \begin{cases} x+5 & \text{if } x > 1 \\ -(x+5) & \text{if } x \le 1 \end{cases}$

$g(x)$ is a piecewise function, i.e., a function that uses two or more rules to find y, depending on the value of x. We first must determine the proper rule to use, then apply it to the given x-value.

To find $g(-3)$, we identify that $x = -3$ is in the second of the two intervals specified, $x \le 1$. So:

$g(x) = -(x+5)$
$g(-3) = -(-3+5) = \mathbf{-2}$

9) Find $h(3)$ if $h(x) = \begin{cases} \frac{x^2+4}{x-3} & \text{if } x \ne 3 \\ x-1 & \text{if } x = 3 \end{cases}$

To find $h(3)$, we identify that $x = 3$ is in the second of the two intervals specified. So:

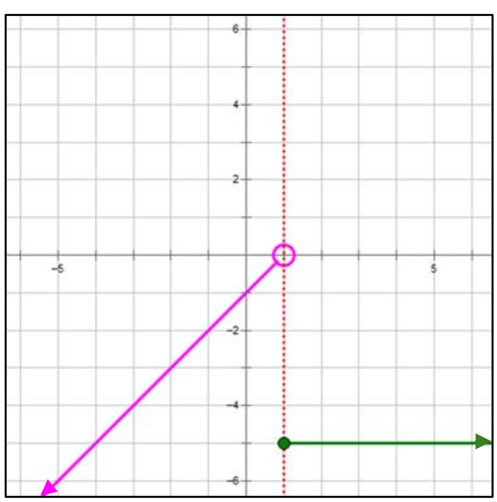
$h(x) = x - 1$
$h(3) = 3 - 1 = \mathbf{2}$

For #10 – 11: Graph each piecewise function.

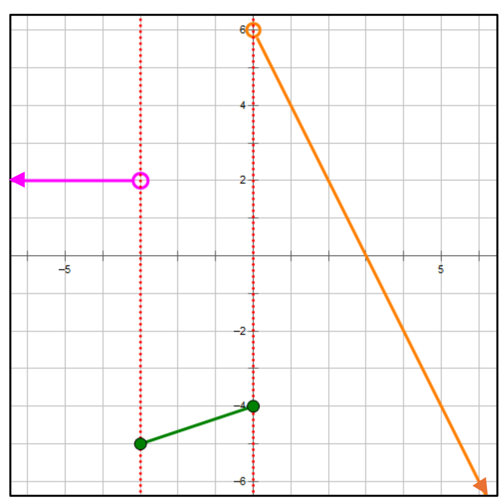
When graphing piecewise functions, it may be helpful to draw vertical dashed or dotted lines at x-values where the function definition changes. By using a line that is dashed or dotted, we are indicating that the line is not part of the solution, merely a help in plotting the piecewise function.

Pay special attention to open and closed circles at the endpoints of the intervals, and to arrows which signify that the function continues forever in the direction indicated.

10) $f(x) = \begin{cases} x-1 & \text{if } x < 1 \\ -5 & \text{if } x \ge 1 \end{cases}$



11) $g(x) = \begin{cases} 2 & \text{if } x < -3 \\ \frac{1}{3}x - 4 & \text{if } -3 \le x \le 0 \\ -2x+6 & \text{if } x > 0 \end{cases}$



12) Given that $f(x) = x^2 + 7x + 5$, find $\frac{f(x+h)-f(x)}{h}$ given that $x \neq 0$. Simplify your answer fully.

We are asked to find $\frac{f(x+h)-f(x)}{h}$, an expression called the difference quotient. The difference quotient provides the slope of a curve over the interval $[x, x+h]$ and will be seen again in Chapter 11, an Introduction to Calculus.

$$\frac{f(x+h)-f(x)}{h} = \frac{[(x+h)^2 + 7(x+h) + 5] - [x^2 + 7x + 5]}{h}$$

$$= \frac{[x^2 + 2hx + h^2 + 7x + 7h + 5] - [x^2 + 7x + 5]}{h}$$

$$= \frac{2hx + h^2 + 7h}{h} = \mathbf{2x + h + 7}$$

For #13 – 14, write the equation of each line in point-slope form, slope-int form, *and* in (h, k) form.

13) Passes through $(1, -4)$ with an x-intercept at $x = -1$.

We have two points on the line, $(1, -4)$ and $(-1, 0)$, so we can find the slope. $(-1, 0)$ is the x-intercept.

$$m = \frac{0-(-4)}{-1-1} = \frac{4}{-2} = -2$$

For **Point-slope form,** use the slope and a point $(-1, 0)$. You can use either point.

$$y - 0 = -2\big(x - (-1)\big) \quad \Rightarrow \quad \mathbf{y = -2(x+1)}$$

(h, k) **form** is similar; use the slope and a point $(-1, 0)$. You can use either point.

$$y = -2\big(x - (-1)\big) + 0 \quad \Rightarrow \quad \mathbf{y = -2(x+1)}$$

To get **slope-intercept** form, expand the right side of the (h, k) form of the equation.

$$y = -2(x+1) \quad \Rightarrow \quad \mathbf{y = -2x - 2}$$

14) Passes through $(8, -7)$ and $(-4, -1)$

We have two points on the line, $(8, -7)$ and $(-4, -1)$, so we can find the slope.

$$m = \frac{-1-(-7)}{-4-8} = \frac{6}{-12} = -\frac{1}{2}$$

For **Point-slope form,** use the slope and a point $(-4, -1)$. You can use either point.

$$y - (-1) = -\frac{1}{2}\big(x - (-4)\big) \quad \Rightarrow \quad \mathbf{y + 1 = -\frac{1}{2}(x+4)}$$

(h, k) **form** is similar; use the slope and a point $(-4, -1)$. You can use either point.

$$y = -\frac{1}{2}\big(x - (-4)\big) + (-1) \quad \Rightarrow \quad \mathbf{y = -\frac{1}{2}(x+4) - 1}$$

To get **slope-intercept** form, expand the right side of the (h, k) form of the equation.

$$y = -\frac{1}{2}(x+4) - 1 \quad \Rightarrow \quad \mathbf{y = -\frac{1}{2}x - 3}$$

For #15 – 19, given that $f(x) = 8x^2 - 5x$ and $g(x) = 3x - 6$, find the requested value or expression.

15) Find $f(x) + g(x)$.

$f(x) + g(x) = (8x^2 - 5x) + (3x - 6) = \mathbf{8x^2 - 2x - 6}$

16) Find $f(x) \cdot g(x)$.

$f(x) \cdot g(x) = (8x^2 - 5x) \cdot (3x - 6) = 24x^3 - 48x^2 - 15x^2 + 30x = \mathbf{24x^3 - 63x^2 + 30x}$

17) Find $\frac{f(x)}{g(x)}$

$$\frac{f(x)}{g(x)} = \frac{8x^2 - 5x}{3x - 6} = \frac{x(8x - 5)}{3(x - 2)}$$

18) Find $f\big(g(x)\big)$. In other words, find $(f \circ g)(x)$.

$$\begin{aligned} f\big(g(x)\big) = f(3x - 6) &= 8(3x - 6)^2 - 5(3x - 6) \\ &= 8(9x^2 - 36x + 36) - 5(3x - 6) \\ &= 72x^2 - 288x + 288 - 15x + 30 \\ &= \mathbf{72x^2 - 303x + 318} \end{aligned}$$

19) Find $f(g(11))$. In other words, find $(f \circ g)(11)$.

We can either substitute 11 into the equation determined in problem 18 or do the calculation in two parts, which is often easier.

$g(11) = 3(11) - 6 = 27$

$f\big(g(11)\big) = f(27) = 8(27)^2 - 5(27) = \mathbf{5697}$

For #20 – 21, Multiple Choice. Select the best answer.

20) Find the functions f and g so that $h(x) = (f \circ g)(x)$ given that $h(x) = \frac{6}{\sqrt{2x+9}}$

A) $f(x) = \sqrt{2x + 9}; g(x) = 6$ B) $f(x) = \frac{6}{x}; g(x) = 2x + 9$

C) $f(x) = 6; g(x) = \sqrt{2 + 9}$ (D) $f(x) = \frac{6}{\sqrt{x}}; g(x) = 2x + 9$

Compositions in each of the possible solutions would give:

A) $f\big(g(x)\big) = \sqrt{2(6) + 9}$ B) $f\big(g(x)\big) = \frac{6}{2x+9}$

C) $f\big(g(x)\big) = 6$ D) $f\big(g(x)\big) = \frac{6}{\sqrt{2x+9}}$ ✓

The composition we are looking for is **Answer D**.

21) Which two functions below are inverses of each other?

$f(x) = \frac{x+3}{2}$ $\quad g(x) = 2x + 3$ $\quad h(x) = \frac{x-3}{2}$

A) $f(x)$ and $h(x)$ (B) $g(x)$ and $h(x)$ C) $f(x)$ and $g(x)$ D) None

If two functions $u(x)$ and $v(x)$ are inverses, then: $u(v(x)) = v(u(x)) = x$.

Tip for finding inverses: Often, inverses will have opposite operations from each other, i.e., if one function contains addition and division, the other will contain subtraction and multiplication.

With that tip in mind, we look over the three functions above and find that $g(x)$ and $h(x)$ have opposite operations. Let's try them:

$$g(x) = 2x + 3;\ h(x) = \frac{x-3}{2} \quad \Rightarrow \quad g(h(x)) = 2\left(\frac{x-3}{2}\right) + 3 = (x-3) + 3 = x$$

This is sufficient work for a multiple choice question when the functions are relatively uncomplicated.

Conclude: **$g(x)$ and $h(x)$ are inverses.** **Answer B.**

22) Find the inverse of the one-to-one function: $f(x) = \frac{7x+8}{5}$

To find the inverse of a function, switch the x's and y's and solve for the new y. Rename functions as necessary.

Start: $y = \frac{7x+8}{5}$

Switch x and y: $x = \frac{7y+8}{5}$

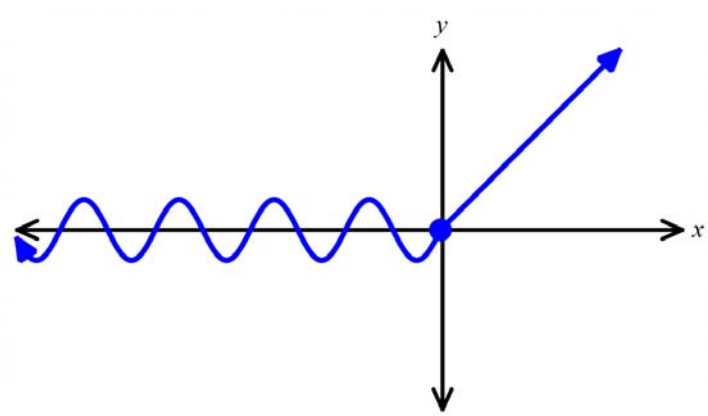
Solve for y: $5x = 7y + 8$

$5x - 8 = 7y$

$\frac{5x-8}{7} = y$

Finally: $f^{-1}(x) = \frac{5x-8}{7}$

23) Does the graph represent a function that has an inverse function? Explain your reasoning.



No, it does not.

Just as a vertical line test can be used to see if a graph represents a function, a horizontal line test can be used to see if a function is one-to-one. Only one-to-one functions have inverse functions. This graph represents a function, but fails the horizontal line test, so it is not one-to-one and has no inverse function.

24) Graph f^{-1} given $f(x)$ as shown. Also, find the domain and range of f^{-1} in interval notation.

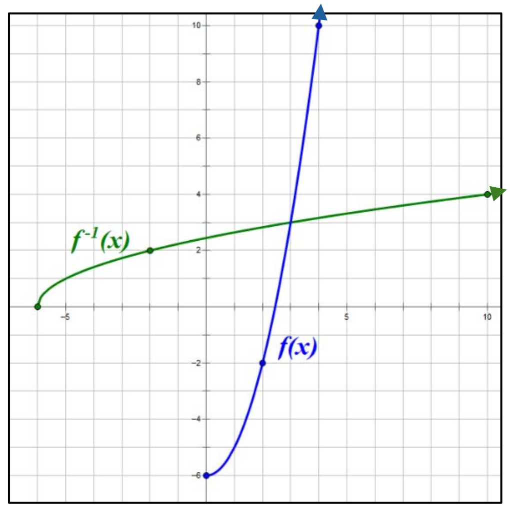
Part 1: To plot an inverse, select points on the original function, then exchange the x- and y-values to obtain points on the inverse function.

Original point: $(0,-6)$ Inverse point: $(-6,0)$

Original point: $(2,-2)$ Inverse point: $(-2,2)$

Original point: $(4,10)$ Inverse point: $(10,4)$

Plot the new points and run a smooth curve through them.



Part 2: Domains and ranges exchange places for inverse functions.

For the function $f(x)$:

Domain: $[0,\infty)$

Range: $[-6,\infty)$

For the function $f^{-1}(x)$, switch the domain and range:

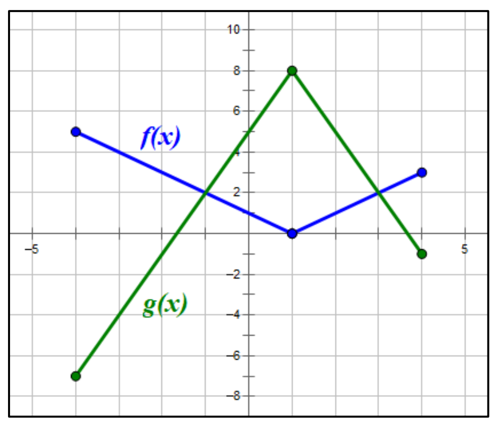
Domain: $[-6,\infty)$

Range: $[0,\infty)$

25) Given $f(x)=4x^2+2x+8$ and $g(x)=2x-3$, find $f\big(g(x)\big)$. In other words, find $(f\circ g)(x)$.

$$f\big(g(x)\big)=f(2x-3)=4(2x-3)^2+2(2x-3)+8$$
$$=4(4x^2-12x+9)+2(2x-3)+8$$
$$=16x^2-48x+36+4x-6+8$$
$$=\mathbf{16x^2-44x+38}$$

26) Given the graph of $f(x)$ as shown below. Graph $g(x)$ on the same coordinate system if $g(x)=-3f(x)+8$.



Since the function consists of straight lines and $g(x)$ is linear in $f(x)$, we can calculate points on $g(x)$ associated with the endpoints of the segments of $f(x)$, and connect them to obtain the graph of $g(x)$.

On the graph provided (blue), we have segment endpoints: $(-4,5)$, $(1,0)$, $(4,3)$. Note the different scales on the x- and y-axes. We need to run each of these endpoints through the definition of $g(x)$ to obtain the new endpoints.

Original point: $(-4,5)$ New y-value: $y=-3(5)+8=-7$ New segment endpoint: $(-4,-7)$

Original point: $(1,0)$ New y-value: $y=-3(0)+8=8$ New segment endpoint: $(1,8)$

Original point: $(4,3)$ New y-value: $y=-3(3)+8=-1$ New segment endpoint: $(4,-1)$

Finally, plot and connect the new points on the graph (green).

27) Graph $y = -2\sqrt{x+3}$ below.

Notice that the domain of this function is $[-3, \infty)$. The negative sign in front means this particular function will be below the x-axis, (except at $x = -3$). Beyond that, it behaves like a typical square root function.

Points for graphing (note that x-values were carefully selected to result in perfect squares under the radical; this is not required, but makes the calculations easier):

$x = -3,\ \ y = -2\sqrt{-3+3} = -2\sqrt{0} = 0,\quad$ Point: $(-3, 0)$

$x = 1,\ \ y = -2\sqrt{1+3} = -2\sqrt{4} = -4,\quad$ Point: $(1, -4)$

$x = 6,\ \ y = -2\sqrt{6+3} = -2\sqrt{9} = -6,\quad$ Point: $(6, -6)$

Run a smooth curve through these points and beyond.